We now make an estimate of the ratio λ/η (I), from equation (I.3.19), which relates the high temperature elastic constant to our experimentally determined values of σ_0 and Δ_0. Using typical values of these parameters from Table 1, we find

$$\tfrac{1}{2}(c_{11}-c_{12}) = 12{\cdot}5\times 10^{11}\left(\frac{1}{1+\dfrac{\lambda}{\eta}}\right)\text{dyn/cm}^2. \tag{4.7}$$

Recent measurements on the elastic constant $c_{11}-c_{12}$ of $DyVO_4$ by Melcher and Scott[21] and by Sandercock[22] show the anomaly at T_d predicted in paper I, and obtain a value at high temperatures of $c_{11}-c_{12}/2 = 9{\cdot}3\pm 0{\cdot}4\times 10^{11}$ dynes/cm^2. Using this value in equation (4.7) gives $\lambda/\eta \approx \frac{1}{3}$ and since $\Delta_0 = \lambda+\eta = 27\ \text{cm}^{-1}$ we obtain $\lambda \approx 7\ \text{cm}^{-1}$ and $\eta \approx 20\ \text{cm}^{-1}$. This result would indicate that the coupling via strain is the dominant interaction driving the transition.

Short range order

As is clear from Fig. 5, the stress data at temperatures near T_D deviates significantly from the molecular field curve even at high stress. We have attributed these deviations to short range order and have obtained a fit using the corrected stress equation (I.3.22). A value of $\Gamma = 4$ gives remarkable agreement with the data, especially considering that it accounts properly for both the temperature-dependence and the stress dependence of the deviations, provided the splitting is sufficiently large.

The observed transition temperature also deviates from the result predicted by molecular field theory[16, 23],

$$\frac{T_d\,(\text{observed})}{T_d\,(\text{molecular field})} = 0{\cdot}74\pm 0{\cdot}02.$$

The simple theory discussed in I which takes account of the first order effects of fluctuation predicts

$$\frac{T_d\,(\text{calculated})}{T_d\,(\text{molecular field})} = \frac{\Gamma}{1+\Gamma}$$

which is again in agreement with experiment for $\Gamma = 4$. The specific heat is strongly affected by the high temperature tail of the Schottky anomaly associated with the splitting $\Delta_\infty = 9\ \text{cm}^{-1}$. It is thus difficult to obtain meaningful results from the specific heat curve on the short range order.

We defined Γ in (I 3.23) as

$$\Gamma = \Big[\sum_m J'(n-m)\Big]^2 \Big/ \Big[\sum_m (J'(n-m))^2\Big].$$

A low value of Γ can occur if either all the interactions are short range (in this case $\Gamma = z$ the number of neighbours) or if there are interactions whose sign changes as a function of angle or distance (such as dipolar forces). The dominant interaction has been shown to be coupling via the strain which is long range. This rules out the first interpretation for Γ and suggests that a spacially oscillating interaction occurs. We showed in I that dipole and higher order multipole interactions are associated with the same coupling coefficient as that which gives rise to the coupling to uniform strain. It is possible that coupling via the optic phonons as well as these multipole interactions are responsible for the fluctuation effects. An independent test of the importance of long range couplings may be obtained from measurements of the ordering temperature as a function of dilution; such measurements are in progress.

The success of the simple molecular field theory and the first order theory for short range order in accounting for the cooperative Jahn–Teller transition in $DyVO_4$ make this system a model one for the study of such phenomena. An understanding of the accidental degeneracy and of the unusually large electron-lattice coupling which are responsible for the transition must await further study of the crystal field and of the actual internal structure of $DyVO_4$.

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